ON COHERENT OPTICAL PULSE PROPAGATION IN ONE-DIMENSIONAL BRAGG GRATING

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ABSTRACT

The propagation of the solitary waves in the Bragg grating formed by array of thin dielectric films is considered. We assume that the thin films of contain the resonant molecules, which are evolved according to two-level atoms model, which is used to description of the coherent optical pulses propagation. There the alternative derivation of the Mantsyzov's equations is represented.

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1 Introduction

In during long time a special kind of gap material has been investigated [1] -[10]. That was referred to as *resonant Bragg grating* [1]-[5] or *resonantly absorbing Bragg reflector* (RABR) [6, 7, 8]. In the simplest case RABR consist of the linear homogeneous dielectric medium containing array of thin films of resonant atoms or molecules. The distance between neighboring films is a, the depth of film (l_f) is great less than wave length of radiation spreading in this structure. The films of two-laves interacted with ultra-short optical pulse have been considered and investigated in framework of the two-wave reduced Maxwell-Bloch equations by Mantsyzov at.al. [1]-[5], and by Kozhekin at.al. [6, 7, 9]. The existence of the 2π-pulse of self-induced transparency was demonstrated [1, 4, 6]. It was found [8] that the bright as well as the dark solitons can exist in the spectral gap, and the bright solitons can have arbitrary pulse area. If the two-level atoms density is very high, then the near-dipole-dipole interaction should be included in the Maxwell-Bloch equations. The effect of dipole-dipole interaction on the existence of gap solitons in the RABR was studied in [10]. The many results of these reviewed in [9]. Recently by using numerical simulation the *optical zoomeron* [14] was discovered and investigated. Optical zoomeron is similar to soliton, however it has the velocity which oscillates near some average value.

The two-wave reduced Maxwell-Bloch equations are the base of the investigation of the solitary wave propagation in the RABR. They have been

deduced by according to coupled-mode theory. There the alternative method of derivation of the evolutions equations describing the ultrashort optical pulses propagation in RABR was considered. Hereafter we follow the works [1]-[8] reasoned that the Bragg resonance corresponds for $a = (\lambda/2)m$ where $m = 1, 2, 3, \ldots$. It will be shown that the exact equations in the form of discrete (recurrent) relations can be obtained. The number of approximations after that leads to the Mantsyzov's equations. The two-wave reduced Maxwell-Bloch equations in more general form [6]-[10] can be deduced by the same procedure also.

2 Transfer-operator approach

Let us consider the ultra-short optical pulse propagation along the X-direction of the periodical array of thin films which are replaced in points $\ldots x_{n-1}, x_n$, $x_{n+1} \ldots$ (Fig.1). The medium between films characterized by the dielectric permittivity ε. Hereafter for the sake of definiteness the TE-wave which has the component of the electric field parallel to the layers will be considered. All results can be easily generalized for the case of the TM-polarized waves.

It is suitable the electric and magnetic strengths $\overrightarrow{E}$, $\overrightarrow{H}$, and polarization of the two-level atoms ensemble $\overrightarrow{P}$ represent in the form of Fourier integrals

$$\overrightarrow{E}(x, z, t) = (2\pi)^{-2} \int\limits_{-\infty}^{\infty} \exp[-i\omega t + i\beta z] \overrightarrow{E}(x, \beta, \omega) dt dz,$$

$$\overrightarrow{H}(x, z, t) = (2\pi)^{-2} \int\limits_{-\infty}^{\infty} \exp[-i\omega t + i\beta z] \overrightarrow{H}(x, \beta, \omega) dt dz,$$

$$\overrightarrow{P}(x_n, z, t) = (2\pi)^{-2} \int\limits_{-\infty}^{\infty} \exp[-i\omega t + i\beta z] \overrightarrow{P}(x_n, \beta, \omega) dt dz.$$

Outside off the films the Fourier components of the vectors $\overrightarrow{E}(x, \beta, \omega)$ and $\overrightarrow{H}(x, \beta, \omega)$ are defined by the Maxwell equations. At points x_n these values are defined from the continuity conditions. Thus, the TE-wave propagation can be considered in framework of the following system

$$\frac{d^2E}{dx^2} + \left(k^2\varepsilon - \beta^2\right) E = 0, \tag{1}$$

$$H_x = -(\beta/k)E, \quad H_z = -(i/k)dE/dx, \quad E_y = E,$$

with boundary conditions [11, 12]

$$E(x_n - 0) = E(x_n + 0), \quad H_z(x_n + 0) - H_z(x_n - 0) = 4i\pi k P_y(x_n, \beta, \omega), \tag{2}$$

where $k = \omega/c$. The solutions of equation (1) in the intervals $x_n < x < x_{n+1}$ can be written as

$$E(x, \beta, \omega) = A_n(\beta, \omega) \exp[iq(x - x_n)] + B_n(\beta, \omega) \exp[-iq(x - x_n)],$$

and

$$H_z(x, \beta, \omega) = qk^{-1} \left\{ A_n(\beta, \omega) \exp[iq(x - x_n)] - B_n(\beta, \omega) \exp[-iq(x - x_n)] \right\},$$

where $q = \sqrt{k^2\varepsilon - \beta^2}$. Hence, the amplitudes A_n and B_n completely determine the electromagnetic field in RABR. Let us consider the point x_n. Electric field at $x = x_n - \delta$ ($\delta << a$) is defined by amplitudes $A_n^{(L)}$ and $B_n^{(L)}$, the field at $x = x_n + \delta$ is defined by $A_n^{(R)}$ and $B_n^{(R)}$. Continuity conditions (1) result in the following relations between these amplitudes

$$A_n^{(R)} + B_n^{(R)} = A_n^{(L)} + B_n^{(L)},$$

$$A_n^{(R)} - B_n^{(R)} = A_n^{(L)} - B_n^{(L)} + 4\pi i k^2 q^{-1} P_{S,n},$$

where $P_{S,n} = P_S(A_n^{(R)} + B_n^{(R)})$ is the surface polarization of thin film at point x_n, which is induced by the electrical field inside film. Thus one can find

$$A_n^{(R)} = A_n^{(L)} + 2\pi i k^2 q^{-1} P_{S,n}, \quad B_n^{(R)} = B_n^{(L)} - 2\pi i k^2 q^{-1} P_{S,n}. \tag{3}$$

With taking account for the strength of electric field outside off the films we can write

$$A_{n+1}^{(L)} = A_n^{(R)} \exp(iqa), \quad B_{n+1}^{(L)} = B_n^{(R)} \exp(-iqa). \tag{4}$$

If the vectors $\psi_n^{(L)} = (A_n^{(L)}, B_n^{(L)})$ and $\psi_n^{(R)} = (A_n^{(R)}, B_n^{(R)})$ are introduced, then the relations (3) can be represented in following form

$$\psi_n^{(R)} = \widehat{U}_n \psi_n^{(L)},$$

where $\widehat{U}_n$ is transfer operator of vector $\psi_n^{(L)}$ through film replaced at point x_n. In general case it is nonlinear operator. The relations (4) are represented by vectorial form

$$\psi_{n+1}^{(L)} = \widehat{V}_n \psi_n^{(R)},$$

where linear operator of the transferring of the vector $\psi_n^{(R)}$ through clearance between adjacent thin films $\widehat{V}_n$ is represented by diagonal matrix

$$\widehat{V}_n = \begin{pmatrix} \exp(iqa) & 0 \\ 0 & \exp(-iqa) \end{pmatrix}.$$

By this means we define the nonlinear transfer-operator of the vector $\psi_n^{(L)}$ through elementary cell of RABR:

$$\psi_{n+1}^{(L)} = \widehat{V}_n \widehat{U}_n \psi_n^{(L)} = \widehat{T}_n \psi_n^{(L)}. \tag{5}$$

In the case of linear medium transfer-operator is frequently used under consideration of the one dimensional photonic crystals, in particular, the distributed feedback structures [13].

In (5) upper index can be omitted, and the equation can be rewritten in form of the following recurrent relation

$$A_{n+1} = A_n \exp(iqa) + 2\pi i k^2 q^{-1} P_{S,n} \exp(iqa), \tag{6}$$

$$B_{n+1} = B_n \exp(-iqa) - 2\pi i k^2 q^{-1} P_{S,n} \exp(-iqa) \tag{7}$$

These recurrent relations are exact equations due to we does not use any approximation (for example, the slowly varying envelope of electromagnetic pulses approximation, the long-wave approximation). Furthermore, the surface polarization of thin film could be calculated on base of deferent suitable models. Here we will follow the works by B. Mantsyzov at al. [1]-[5], and A.Kozhekin, G.Kurizki at.al. [6]-[9], where the two-levels atom model has been used.

3 Linear response approximation

To demonstrate that the RABR is real gap media it is suitable to obtain the electromagnetic wave spectrum in linear response approximation. In the general case we can use the following expression for polarization

$$P_{S,n} = \chi(\omega)(A_n^{(R)} + B_n^{(R)}) \tag{8}$$

Substitution of this formula in (6),(7) leads to

$$A_{n+1} = (1 + i\rho)A_n \exp(iqa) + i\rho B_n \exp(iqa), \tag{9}$$

$$B_{n+1} = (1 - i\rho) B_n \exp(-iqa) - i\rho A_n \exp(-iqa). \tag{10}$$

Here $\rho = \rho(\omega) = 2\pi k^2 q^{-1} \chi(\omega) = 2\pi\omega c^{-1} \varepsilon^{1/2} \chi(\omega)$. The wave as collective motion of the electrical field in grating corresponds to anzats

$$A_n = A \exp(ikna), \qquad B_n = B \exp(ikna). \tag{11}$$

From (9),(10) it follows the linear equations are defining the amplitudes of the wave A and B:

$$A \exp(ika) = (1 + i\rho) A \exp(iqa) + i\rho B \exp(iqa), \tag{12}$$

$$B \exp(ika) = (1 - i\rho) B \exp(-iqa) - i\rho A \exp(-iqa). \tag{13}$$

Nontrivial solution of this system is exists only if the determinant is not zero, i.e.,

$$\det \begin{pmatrix} (1 + i\rho) \exp(iqa) - \exp(ika) & i\rho \exp(iqa) \\ -i\rho \exp(-iqa) & (1 - i\rho) \exp(-iqa) - \exp(ika) \end{pmatrix} = 0. \tag{14}$$

Let us define

$$Z = \exp(ika),$$

$$G = (1 + i\rho) \exp(iqa) = (\cos qa - \rho \sin qa) + i(\rho \cos qa + \sin qa)$$

Then the equation (14) can be rewritten as equation in Z:

$$Z^2 - (G + G^*) Z + 1 = 0.$$

The solution of that is

$$Z_\pm = \operatorname{Re} G \pm i\sqrt{1 - (\operatorname{Re} G)^2}.$$

If $\mathrm{Re}\, G \leq 1$, then $\mathrm{Re}\, G \pm i\sqrt{1 - (\mathrm{Re}\, G)^2} = \cos ka + i \sin ka$. Hence the wave numbers $k_\pm$ are real values and they are defined from transcendent equation

$$\cos ka = \cos qa - \rho \sin qa. \tag{15}$$

That is the dispersion relation which defines the dependence of wave number $k_\pm$ versus frequency of harmonic wave propagating in linear RABR.

If $\mathrm{Re}\, G > 1$, then the roots of equation (14) are real ones. It means that wave numbers $k_\pm$ are imaginary values. The condition $\mathrm{Re}\, G > 1$ defines the frequencies of the forbidden zone. The waves with these frequencies can not propagate in grating. Thus the boundaries of this forbidden zone follow from the equation

$$\cos qa - \rho \sin qa = 1. \tag{16}$$

Model of the resonant system containing the thin films defines the explicit form of the function $\rho = \rho(\omega)$, that results in the different dispersion relations (15). It should be pointed out that this dispersion relation demonstrates a series of gaps in the electromagnetic wave spectrum.

4 Long-wave and weak nonlinearity approximations

By using long-wave approximation we can transform the exact equations (6),(7) into the differential equations. To do it is suitable to introduce the field variables

$$A(x) = \sum_n A_n \delta(x - x_n), \quad B(x) = \sum_n B_n \delta(x - x_n),$$

$$P(x) = \sum_n P_{S,n}\delta(x - x_n),$$

Using the integral representation for delta-function

$$\delta(x) = (2\pi)^{-1} \int_{-\infty}^{\infty} \exp(ikx)dk,$$

one can obtain the following expression

$$\begin{aligned} A(x) &= (2\pi)^{-1} \sum_n A_n \int_{-\infty}^{\infty} \exp[ik(x - x_n)]dk = \\ &= (2\pi)^{-1} \int_{-\infty}^{\infty} \exp(ikx) \sum_n A_n \exp(-ikx_n)dk. \end{aligned}$$

From that

$$A(k) = \sum_n A_n \exp(-ikx_n) = \sum_n A_n \exp(-ikan),$$

and by the same way one get

$$B(k) = \sum_n B_n \exp(-ikan), \quad P(k) = \sum_n P_{S,n} \exp(-ikan).$$

The definition of these spatial Fourier components results in the periodicity conditions, for example,

$$\begin{aligned} A(k) &= \sum_n A_n \exp(-ikan) \exp(\pm 2\pi in) = \\ &= \sum_n A_n \exp(-ikan \pm 2\pi in) = \\ &= \sum_n A_n \exp[-ian(k \pm 2\pi/a)] = A(k \pm 2\pi/a). \end{aligned} \tag{17}$$

From recurrent equations (6),(7) we have

$$A(k) \exp(ika) = A(k) \exp(iqa) + i\kappa P(k) \exp(iqa),$$

$$B(k)\exp(ika) = B(k)\exp(-iqa) - i\kappa P(k)\exp(-iqa),$$

or

$$[\exp\{ia(k-q)\} - 1]\, A(k) = i\kappa P(k), \tag{18}$$

$$[\exp\{ia(k+q)\} - 1]\, B(k) = -i\kappa P(k). \tag{19}$$

The linear approximation for polarization $P(k) = \chi(\omega)[A(k) + B(k)]$ leads us to linear dispersion law (14) once again. The system of equations (18),(19) is equivalent of discrete equations (6),(7). At this point we have not approximation, apart from assumption of thin films width.

If the thin film array was absent then the dispersion relation would be

$$\cos ka = \cos qa.$$

From that, for right wave with amplitude $A(k)$ we have $k = q$, whereas for wave with amplitude $B(k)$ propagation in opposite direction we have $k = -q$. Let us suppose that the polarization of the thin film array produce the little change in wave vectors, i.e., for right wave it is $k = q + \delta k$ and for the opposite wave it is $k = -q + \delta k$. Let us chouse the value of q near one of the Bragg resonances, to say, $q = 2\pi/a + \delta q$, where $\delta q \ll 2\pi/a$. In this case the equations (6),(7) take the form

$$[\exp(ia\delta k) - 1]\, A(q + \delta k) = i\kappa P(q + \delta k),$$

$$[\exp(ia\delta k) - 1]\, B(-q + \delta k) = -i\kappa P(-q + \delta k).$$

By using the periodicity conditions (17) these equations are rewritable as

$$[\exp(ia\delta k) - 1]\, A(\delta q + \delta k) = i\kappa P(\delta q + \delta k),$$

$$[\exp(ia\delta k) - 1]\, B(-\delta q + \delta k) = -i\kappa P(-\delta q + \delta k).$$

After changing of the variables $\delta k = \delta\tilde{k} \pm \delta q$ we have

$$\left[\exp\{ia(\delta\tilde{k} - \delta q)\} - 1\right] A(\delta\tilde{k}) = i\kappa P(\delta\tilde{k}), \tag{20}$$

$$\left[\exp\{ia(\delta\tilde{k} + \delta q)\} - 1\right] B(\delta\tilde{k}) = -i\kappa P(\delta\tilde{k}). \tag{21}$$

The long-wave approximation means that nonzero values of the spatial Fourier amplitudes are located near zero value of argument. Hence, we can assume $a\delta\tilde{k} \ll 1$ in exponential functions, that results in the following approximate equations

$$ia(\delta\tilde{k} - \delta q)A(\delta\tilde{k}) = i\kappa P(\delta\tilde{k}), \tag{22}$$

$$ia(\delta\tilde{k} + \delta q)B(\delta\tilde{k}) = -i\kappa P(\delta\tilde{k}). \tag{23}$$

If now we return in to spatial variable, then equations (22),(23) lead us to the equations of coupled wave theory

$$\partial A/\partial x = i\delta q A(x) + i\kappa a^{-1} P(x), \tag{24}$$

$$\partial B/\partial x = -i\delta q B(x) - i\kappa a^{-1} P(x). \tag{25}$$

In these equations the both fields $A(x), B(x), P(x)$ and parameters δq, κ are the functions of frequency ω. To obtain final system of equations in the coordinate and time variables the inverse Fourier transformation would be done. However, frequently the assumption of a slowly varying it time scale envelopes of electromagnetic pulses and polarization used [14]. This approximation allows simplifying the system of coupled wave equations under consideration.

5 Slowly varying envelopes approximations

Slowly varying envelopes approximation minds that we have to deal with narrow wave packets or, what is the same, with quasiharmonic waves [14]. For example, for the quasiharmonic wave the electric field is represented by expression

$$E(x,t) = \mathcal{E}(x,t)\exp[-i\omega_0 t],$$

where ω_0 is carrier wave frequency. The electric field $E(x,t)$ and Fourier components of the envelope of the pulse $\tilde{E}(x,t)$ are related by the following relationship

$$\begin{aligned} E(x,t) &= (2\pi)^{-1}\int\limits_{-\infty}^{\infty} E(x,\omega)\exp(-i\omega t)d\omega = \\ &= (2\pi)^{-1}\int\limits_{-\infty}^{\infty} \mathcal{E}(x,\omega)\exp[-i(\omega+\omega_0)t]d\omega = \\ &= (2\pi)^{-1}\int\limits_{-\infty}^{\infty} \mathcal{E}(x,\omega-\omega_0)\exp(-i\omega t)d\omega, \end{aligned}$$

where the function $E(x,\omega)$ is nonzero one if ω belongs to interval $(\omega_0 - \Delta\omega, \omega_0 + \Delta\omega)$ with $\Delta\omega \ll \omega_0$. It leads to $E(x,\omega+\omega_0) = \mathcal{E}(x,\omega)$. Thus, if we have some relation for $E(x,\omega)$, then in order to carry out the needed relation for $\mathcal{E}(x,\omega)$ to be done shifting $\omega \to \omega_0 + \omega$ in all functions of ω.

Let be

$$A(x,t) = \mathcal{A}(x,t)\exp(-i\omega_0 t), \quad B(x,t) = \mathcal{B}(x,t)\exp(-i\omega_0 t),$$

$$P(x,t) = \mathcal{P}(x,t)\exp(-i\omega_0 t).$$

For the Fourier components $\mathcal{A}(x,\omega), \mathcal{B}(x,\omega)$ and $\mathcal{P}(x,\omega)$ from (24),(25) it follows

$$\partial \mathcal{A}(x,\omega)/\partial x = i\delta q(\omega_0+\omega)\mathcal{A}(x,\omega) + i\kappa(\omega_0+\omega)a^{-1}\mathcal{P}(x,\omega), \tag{26}$$

$$\partial \mathcal{B}(x,\omega)/\partial x = -i\delta q(\omega_0+\omega)\mathcal{B}(x,\omega) - i\kappa(\omega_0+\omega)a^{-1}\mathcal{P}(x,\omega) \tag{27}$$

Inasmuch as in this expression $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{P}$ are distinguished from zero at $\omega \ll \omega_0$, one can use the expansions:

$$\delta q(\omega_0+\omega) \approx q_0 - 2\pi/a + q_1\omega + q_2\omega^2/2, \quad \kappa(\omega_0+\omega)a^{-1} \approx K_0. \tag{28}$$

where $q_n = d^n q/d\omega^n$ at $\omega = \omega_0$. In particular, the $q_1^{-1} = v_g$ is the group velocity, q_2 takes account the group-velocity dispersion.

With taking expansions (28) into account we can write following equations which describe the evolution of slowly varying envelopes

$$i\left(\frac{\partial}{\partial x} + \frac{1}{v_g}\frac{\partial}{\partial t}\right)\mathcal{A} - \frac{q_2}{2}\frac{\partial^2}{\partial t^2}\mathcal{A} + \Delta q_0\mathcal{A} = -K_0\mathcal{P}, \tag{29}$$

$$i\left(\frac{\partial}{\partial x} - \frac{1}{v_g}\frac{\partial}{\partial t}\right)\mathcal{B} + \frac{q_2}{2}\frac{\partial^2}{\partial t^2}\mathcal{B} - \Delta q_0\mathcal{B} = +K_0\mathcal{P}, \tag{30}$$

where $\Delta q_0 = q_0 - 2\pi/a$. To do next step it need to chouse the model for thin film medium. It can be enharmonic oscillators, the two- or three-levels atoms, the excitons of molecular chains, nano-particles, the quantum dots, and so on. Here we consider the two-level atom model.

6 Two level atoms approximations

The two-level atom state defines by the density matrix $\hat{\rho}$. The matrix element ρ_{12} describes the transition between the ground state $|2\rangle$ and excited state

$|1\rangle$, ρ_{22} and ρ_{11} represents the population of theses states. Evolution of the two-level atom is governed by the Bloch equations [15].

$$i\hbar\frac{\partial}{\partial t}\rho_{12} = \hbar\Delta\omega\rho_{12} - d_{12}(\rho_{22} - \rho_{11})A_{in}, \tag{31}$$

$$i\hbar\frac{\partial}{\partial t}(\rho_{22} - \rho_{11}) = 2\left(d_{12}\rho_{21}A_{in} - d_{21}\rho_{12}A^*_{in}\right). \tag{32}$$

In these equations A_{in} is the electric field interacting with two-level atom. In the problem under consideration we have $A_{in} = \mathcal{A} + \mathcal{B}$ and

$$K_0\mathcal{P} = \frac{2\pi\omega_0 n_{at} d_{12}}{cn(\omega_0)}\left\langle\rho_{12}\right\rangle.$$

There the cornerstone brackets denote summation over all atoms within a frequency detuning $\Delta\omega$ from center of the inhomogeneously broadening line, $n(\omega_0)$ is the refractive index of the medium containing the array of thin films, n_{at} is effective density of the resonant atoms in films. This value is defined thought bulk density of atoms N_{at}, film width l_f and lattice spacing by formula $n_{at} = N_{at}(l_f/a)$.

Let us suppose the group-velocity dispersion is of no importance. The resulting equations are the *two-wave reduced Maxwell-Bloch equations*. It is suitable to introduce the normalized variables

$$e_1 = t_0 d_{12}\mathcal{A}/\hbar, \quad e_2 = t_0 d_{12}\mathcal{B}/\hbar, \quad x = \zeta v_g t_0, \quad \tau = t/t_0.$$

The normalized two-wave reduced Maxwell-Bloch equations take the following form

$$i\left(\frac{\partial}{\partial\zeta} + \frac{\partial}{\partial\tau}\right)e_1 + \delta e_1 = -\gamma\left\langle\rho_{12}\right\rangle, \tag{33}$$

$$i\left(\frac{\partial}{\partial\zeta} - \frac{\partial}{\partial\tau}\right)e_2 - \delta e_2 = +\gamma\left\langle\rho_{12}\right\rangle, \tag{34}$$

$$i\frac{\partial}{\partial\tau}\rho_{12} = \Delta\rho_{12} - ne_{in}, \tag{35}$$

$$\frac{\partial}{\partial\tau}n = -4\,\mathrm{Im}(\rho_{12}e^*_{in}), \tag{36}$$

where $\gamma = t_0 v_g/L_a$, $\delta = t_0 v_g \Delta q_0$, $L_a = (cn(\omega_0)\hbar)/(2\pi\omega_0 t_0 n_{at}|d_{12}|^2)$ is resonant absorption length and $\Delta = \Delta\omega t_0$ is normalized frequency detuning. We use the following variables $n = \rho_{22} - \rho_{11}$, $e_{in} = e_1 + e_2$.

Let define new variables according to following expressions

$$e_{in} = e_1 + e_2 = f_s \exp(i\delta\tau), \quad e_1 - e_2 = f_a \exp(i\delta\tau), \quad \rho_{12} = r\exp(i\delta\tau).$$

The system of equations (33) - (36), can be rewritten as

$$\frac{\partial f_s}{\partial\zeta} + \frac{\partial f_a}{\partial\tau} = 0, \tag{37}$$

$$\frac{\partial f_a}{\partial\zeta} + \frac{\partial f_s}{\partial\tau} = 2i\gamma\left\langle r\right\rangle, \tag{38}$$

$$i\frac{\partial}{\partial\tau}r = (\Delta+\delta)r - nf_s, \tag{39}$$

$$\frac{\partial}{\partial\tau}n = -4\,\mathrm{Im}(rf_s^*). \tag{40}$$

From (37) it follows

$$\frac{\partial f_a}{\partial\zeta} = -\frac{\partial f_s}{\partial\tau},$$

It allows rewrite the system (37)-(40) in another form:

$$\frac{\partial^2 f_s}{\partial\zeta^2} - \frac{\partial^2 f_s}{\partial\tau^2} = -2i\gamma\left\langle\frac{\partial r}{\partial\tau}\right\rangle, \tag{41}$$

$$i\frac{\partial}{\partial\tau}r = (\Delta+\delta)r - nf_s, \tag{42}$$

$$\frac{\partial}{\partial\tau}n = -4\,\mathrm{Im}(rf_s^*) \tag{43}$$

If we assume that the inhomogeneous broadening is absent, i.e., the hypothesis of a sharp atomic resonant transition is true, this system is reduced to Sine-Gordon equation $\delta + \Delta = 0$ [1]. In [4] the steady state solution of (41)-(43) with taking inhomogeneous broadening into account was found.

7 Conclusion

There the derivation of the equations which are describing the ultrashort pulses propagation in one dimensional resonant Bragg grating (or RABR) was considered. The grating consists in thin films array embedded into linear dialectical medium. The exact discrete equation for amplitudes electric field inside films was obtained. The long-wave approximation and slowly varying envelope of the pulses approximation result in the system of the coupled wave equations. At this point it need to chouse the model for thin film medium. In [1]- [10] the two-level atom model has been used. However, other models could be considered that results in new kind of RABRs. For example, array of thin films of the resonant optical materials with embedded metal nanostructures [16]. would be described by the following system

$$i\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\tau}\right)e_1+\delta e_1=-\gamma\left\langle\sigma\right\rangle, \tag{44}$$

$$i\left(\frac{\partial}{\partial\zeta}-\frac{\partial}{\partial\tau}\right)e_2-\delta e_2=+\gamma\left\langle\sigma\right\rangle, \tag{45}$$

$$i\frac{\partial}{\partial\tau}\sigma-\Delta\sigma+\mu|\sigma|^2\sigma=(e_1+e_2). \tag{46}$$

where μ is coupling constant associated with enharmonicity of the plasmonic oscillations. Second example of the resonant model corresponds with three-level atom and two-frequency electromagnetic field. Resulting system of the

basic equations is similar to system considered in [17], where the propagation of the polarized waves in RABR has been studied. A fascinating model was proposed by A. Zabolotskii [18], where thin films are replaced with J-aggregates. Array of thin ferroelectric films represent the gap-medium where the switching wave propagation could be expected.

It should be noted that in [6] - [9] the two-wave reduced Maxwell-Bloch equations contain extra terms taking account the dispersion of polarization. These terms can be provided by this analysis also.

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FIGURE CAPTIONS

Fig. 1. Model of periodical nonlinear medium corresponding with resonantly absorbing Bragg reflector.